

# Syllabus

## Applied Mathematics and Statistics 553.721

### Probability Theory II - Spring 2024

#### Description

This course is an advanced graduate-level course designed to deepen students' understanding of the theoretical underpinnings of probability. Building on the foundational concepts introduced in Probability Theory I, this course delves into more complex topics such as martingales, Markov chains, Brownian motion, and stochastic calculus. Through rigorous mathematical proofs and problem-solving exercises, students will explore the interplay between theory and application, gaining insights into how probability theory informs various fields such as finance, physics, and statistics. The course aims to equip students with a solid theoretical background and analytical skills necessary for research and application in their respective disciplines.

#### Coordinates

**Time:** MW 1:30PM - 2:45PM

**Location:** Maryland 217

#### Personnel

##### Instructor

Mateo Díaz (mateodd@jhu.edu)

Office: Wyman S429

OH: Monday 3:00PM - 5:00PM

##### Teaching Assistants

Ao Sun (asun17@jhu.edu)

OH: Friday 9:00AM - 11:00AM Wyman S425

#### Textbooks

We will use the following references:

- [D] (**Main textbook**) Rick Durrett, Probability: Theory and Examples, 5th edition. Cambridge University Press (2019). Available at [https://services.math.duke.edu/~rtd/PTE/PTE5\\_011119.pdf](https://services.math.duke.edu/~rtd/PTE/PTE5_011119.pdf).
- [B] Patrick Billingsley, Probability and Measure, Anniversary (or 3rd) edition. John Wiley & Sons (2012). Available online through JHU libraries: <https://ebookcentral.proquest.com/lib/jhu/detail.action?docID=836625>.
- [C] Kai Lai Chung, A Course in Probability Theory. Academic Press (2001).
- [W] David Williams, Probability with Martingales, 1st Edition. Cambridge Mathematical Textbooks (1991).

## Topics

- Characteristic functions, further topics (D3.3.4-5, C6.5): Moment problem, Bochner's Theorem, Polya's criterion, stable laws.
- Poisson process (D3.6-7, B23): Construction and properties, Poisson convergence.
- Conditional probability and expectation (B32-33, D A.4, 4.1).
- Martingales (D4, B35): (sub/super)martingales, martingale convergence theorem, Doob's decomposition, Doob's inequality, convergence in  $L^p$ , uniform integrability and convergence in  $L^1$ , backwards martingales, stopping times, optional stopping theorem.
- Kolmogorov's extension theorem (B36, D A.3).
- Markov chains (D5): (strong) Markov property, recurrence and transience, stationary measure, convergence to stationarity.
- Random walks on  $\mathbb{R}^d$  (D4.9, throughout D5, D5.4): reflection principle, recurrence and transience, ballot theorem, visits to 0, arcsine law.
- Brownian motion (D7-8): construction, (strong) Markov property, Blumenthal's 0-1 law, path properties (zeros, hitting times, modulus of continuity), martingale properties, (D8) Skorokhod's representation theorem, Donsker's Theorem, Brownian bridge, law of the iterated logarithm.

## Grading

Your grade will take into account four components: **Homework (40%)**, **Take-home exam (25%)**, **Final project (25%)**, and **Participation (10%)**. In what follows we elaborate on each of these components.

### Course Assessment - Homework

Homework assignments (approximately five) will be posted here and on the course Canvas. Some homework assignments include at least one question that involves the writing and testing of code; Python is preferred. Please submit homework assignments on Gradescope.

Your solutions must be written legibly and intelligibly in clear English. Use complete sentences. Points may be taken off for disorganized or illegible work. **Collaboration is welcome, but your writeup must be your own — no copying. Indicate at the top of your homework who you collaborated with on the assignment.** If you believe your homework or exam grade to be in error, submit a regrade request. **Every student will be allowed \*one instance of submitting late homework, up to 24 hours after the due date (no questions asked)\*.**

### Course Assessment - Take-home exams

There will be one take-home exam with a date TBA. The exam will be posted on Canvas, and you will have two dates to turn in your solutions through Gradescope. **You may not discuss the exam with anyone or otherwise seek external help.**

## Course Assessment - Final project

There will be a final project; this is an opportunity for students to learn about related topics that we did not cover in class. Students can work in groups of two or three people. The final project includes two deliverables: a written report and a 25-minute presentation. Topics for the final project will be released two weeks before the end of class (students are welcome to pick a different project provided the approval of the instructor). Each group can choose how to split the time among the group members. In 25 minutes, the group should aim to state one result/idea, place it in context (why is it interesting? What implications does it have? Does it open interesting new research directions?), and present the main idea of its proof.

## Course Assessment - Participation

Participation weights 10% in the final grade. Engaging in class, Piazza, and office hours will count toward participation. This includes asking questions (even if you think they are silly!) and pointing out typos or mistakes.

## Ethics

The strength of the university depends on academic and personal integrity. In this course, you must be honest and truthful. Ethical violations include cheating on exams, plagiarism, reuse of assignments, improper use of the Internet and electronic devices, unauthorized collaboration, alteration of graded assignments, forgery and falsification, lying, facilitating academic dishonesty, and unfair competition.

Report any violations you witness to the instructor. You may consult the associate dean of student conduct (or designee) by calling the Office of the Dean of Students at 410-516-8208 or via email at [integrity@jhu.edu](mailto:integrity@jhu.edu).

## Personal Well-being

If you are sick please notify me by email so that we can make appropriate accommodations should this affect your ability to attend class, complete assignments, or participate in assessments. The Student Health and Wellness Center is open and operational for primary care needs. If you would like to speak with a medical provider, please call 410-516-8270, and staff will determine an appropriate course of action based on your geographic location, presenting symptoms, and insurance needs. Telemedicine visits are available only to people currently in Maryland. See also <https://studentaffairs.jhu.edu/student-life/student-outreach-support/absences-from-class/illness-note-policy/>.

The Johns Hopkins COVID-19 Call Center (JHCCC), which can be reached at 833-546-7546 seven days a week from 7 a.m. to 7 p.m., supports all JHU students, faculty, and staff experiencing COVID-19 symptoms. Primarily intended for those currently within driving distance of Baltimore, the JHCCC will evaluate your symptoms, order testing if needed, and conduct contact investigation for those affiliates who test positive. More information on the JHCCC and testing is on the coronavirus information website. All students with disabilities who require accommodations for this course should contact me at their earliest convenience to discuss their specific needs. If you have a documented disability, you must be registered with the JHU Office for Student Disability Services (385 Garland Hall; 410-516-4720; <http://web.jhu.edu/disabilities/>) to receive accommodations.

Students who are struggling with anxiety, stress, depression or other mental health related concerns, please consider connecting with resources through the JHU Counseling Center. The Counseling Center will be providing services remotely to protect the health of students, staff, and communities. Please reach out to get connected and learn about service options based on where you are living this fall at 410-516-8278 and online at <http://studentaffairs.jhu.edu/counselingcenter/>. Student Outreach & Support will be fully operational (virtually) to help support students. Students can self-refer or refer a friend who may need extra support or help getting connected to resources. To connect with SOS, please email deanofstudents@jhu.edu, call 410-516-7857, or students can schedule to meet with a Case Manager by visiting the Student Outreach & Support website and follow “Schedule an Appointment.”

## **Classroom Climate**

As your instructor, I am committed to creating a classroom environment that values the diversity of experiences and perspectives that all students bring. Everyone here has the right to be treated with dignity and respect. I believe fostering an inclusive climate is important because research and my experience show that students who interact with peers who are different from themselves learn new things and experience tangible educational outcomes. Please join me in creating a welcoming and vibrant classroom climate. Note that you should expect to be challenged intellectually by me, the TAs, and your peers, and at times this may feel uncomfortable. Indeed, it can be helpful to be pushed sometimes in order to learn and grow. But at no time in this learning process should someone be singled out or treated unequally on the basis of any seen or unseen part of their identity. If you ever have concerns in this course about harassment, discrimination, or any unequal treatment, or if you seek accommodations or resources, I invite you to share directly with me or the TAs. I promise that we will take your communication seriously and to seek mutually acceptable resolutions and accommodations. Reporting will never impact your course grade.

## **Family Accommodations Policy**

You are welcome to bring a family member to class on occasional days when your responsibilities require it (for example, if emergency child care is unavailable, or for health needs of a relative). Please be sensitive to the classroom environment, and if your family member becomes uncomfortably disruptive, you may leave the classroom and return as needed.