Lecture 21 Mon Apr/08/2024 Last time Joday
DExistence Brownian Motion DExis'tennce Formal construction Last time we closed with Theorem (0): $\frac{1}{2}$ $\frac{1}{2$ one D Formal construct
Last time we
Theotem (0): M 0
to paths w: Q $\overline{2}$ $\stackrel{|\phi|}{\rightarrow}$ $\begin{array}{lll} \text{mod} & \text{with} & \text{and} & \text{one} & \text{with} & \text{one} & \text{then} & \text$ μ continuous in a_2 . $\overline{\mathcal{A}}$ The proof of this result follows easily from:
Theorem (.): Suppose that $E[Y_s - X_e]^g$
 $\le K|t-s|^{1+\alpha}$ where $\alpha, \beta > 0$. Then
 $f(1, \alpha) < \alpha / a$ then with α bability Theorem (.): Suppose that $E[Y_{5}-X_{\epsilon}]^{\beta}$ \leq K $16-5$ 1^{1+10} where α $\begin{array}{ccc} \n \text{res} & \text{degined on} \\ \n \text{ref} & \text{triv} \\ \n \text{res} & \text{triv} \\ \n \text{res} & \text{triv} \n \end{array}$ if y $\overline{\textbf{C}}$ α/β then with probability one J C(w) so that e^{in}
 $\frac{e^{in}}{K|t-s|^{1+\alpha}}$ where $\frac{\alpha}{\alpha}, \beta >$
 $\gamma < \frac{\alpha}{\beta}$ then with
 $\frac{1}{\alpha} \frac{1}{\alpha} - \frac{\alpha}{\beta}$ and
 $\frac{1}{\alpha} \frac{1}{\alpha} - \frac{\alpha}{\alpha} = \frac{1}{\alpha}$
 $\frac{1}{\alpha} \frac{1}{\alpha} - \frac{\alpha}{\alpha} = \frac{1}{\alpha}$
 $\frac{1}{\alpha} \frac{1}{\alpha} - \frac{\alpha}{\alpha} = \frac{1}{\alpha}$
 $|9-r|^{8}$ $|9-r|^{8}$ $|9-r|^{8}$ $|9-r|^{8}$ $|9-r|^{8}$ - Proof of Theorem (9) : Notice that by our construction $E18_{t}-8_{s}1^{4}$ = $E18_{t-s}1^{4}$ = $E1(t-s)^{1/2}B.1^{4}$ $= (t - 5)^2$ E B⁴ = 3 (b - S)².
= (t - 5)² E B⁴ = 3 (b - S)². \sim $N(0,1)$

Thus involting Theorem (.) gields a.s. \pm 0 s.t. $|B_{t}-B_{s}| \le C |t-s|^{1/2}$ $\forall t_{0} s \in [a_{2}]$ which immediately implies uniform continuity. Proof of Theorem (.): Note that it suffices to show that $|x_4 - x_1| \leq A |q-r|^\gamma$ $\forall q, r \in \mathbb{Z}_2$ s.t. Real valued 19- r 158.
 $r \cdot V$ (4(w), 8(w) >0 (∞) If (∞) holds then $\forall s, t \in Q_z$ we can find $s = s_0 < ... < s_n = t$ s.t $|S_{i}-S_{i-1}| \leq S$ and $|X_{s} - X_{t}| \leq |X_{s} - X_{s}| + |X_{s_{2}} - X_{s}| + \cdots + |X_{t} - X_{s}|$ $4(5-5.1^{\gamma}+\ldots+|\chi_{t}-s_{n}|^{\gamma})$ = A $(15.5)^{r}$ + $\pm 1x_{t}$ - $5a)^{r}$ | $5-6$ |⁸ $15 - 613$ $4 \left(\sum_{i=1}^{r_{0}+1} 1 \right) | 5 - t |^{ \gamma}$ $\leq A [S^{-1}], |S-t|^{d}$. LCW

Thus, we focus on proving (00). Let $S_n = \begin{cases} 1 \times (m/2^n) - \times (m-1/2^n) < 2^{-\gamma n} \\ \text{for all } 1 \leq m \leq 2^n \end{cases}$ By Markov's ineg : Ym us, we focus
f
Sn = 1 × (m/2n)
for all
Markov's inee
x(m/2n) - X (m-22. $P\left[\left|\sqrt[m]{2}\right|-\chi(m-\frac{1}{2n})\right]\geq 2^{\gamma n}\right]\leq \left[\mathbb{E}|\Upsilon_{m}|^{2}\right]2^{\gamma_{n}\beta}$ $=$ K 2^{-n} (1 + α -8B) Thus, taking union bound
 $P(G_n^c) \leq 2^n K2^{-n(1+\alpha-\delta\beta)}$ $\mathcal{Y}(\boldsymbol{\beta}) = K2 - n(\boldsymbol{\alpha}-\boldsymbol{\gamma}\boldsymbol{\beta})$ The proof follows from the following Lemma : Lemma:
Lemma(4)On Hw = $\bigcap_{n=N}^{\infty} G_n$, we have $\max_{\begin{array}{c} |x_{9}-x_{1}| \leq \frac{4}{1-2^{-\delta}} |y-r|^{\delta} & \text{if } q, r \in \mathbb{Q}_{2} \end{array}}$
 $|x_{9}-x_{1}| \leq \frac{4}{1-2^{-\delta}} |y-r|^{\delta}$ $\begin{array}{c} |y_{9}-x| \leq \frac{4}{1-2^{-\delta}} |y-r| \leq \frac{4}{1-2^{-\delta}} \end{array}$ f q, r \in le 2
sd. ly-rl < 2⁻ⁿ $5d \frac{1}{4} - r \frac{1}{6} < 2^{-N}$ we will come back to the proof of this Lemma , but for now note that $P(H_N^c)$ $\leq \sum_{n=1}^{\infty} P(G_n^c) \leq K \sum_{n=N}^{\infty} 2^{-n(\alpha-3\beta)}$ $\sum_{n=N}$ $\binom{n}{N}$ $\binom{n}{N}$ nsN

Since
$$
\gamma < \underline{\alpha}
$$
 $\Rightarrow \sum_{n=1}^{\infty} P(H_n) < \infty$. By
Borell canhelli, H_n^{c} only holds for finite
N, so (so) holds, convolving the
Proof of Theorem (.)

Proof of Lemma (G) : Take $r, q \in Q$
with our - q < 2^{-N}. Define $I_m^n = \binom{m-1}{2^n}$, $m/2^n$. Let n^t = mox $4n$: In s.t qeI_m^n and ref_{m+l}^n . Let's see a picture. \int_{σ^0} $T_{m+1}^{n^+}$ $\begin{array}{ccc} & & & \\ \hline & & & \\ \hline & & & \\ \end{array}$ can write Then $r = m2^{-n^2} + 2^{-r_2} + \cdots + 2^{-r_{\ell}}$
 $q = m2^{-n^2} - 2^{q_1} - \cdots - 2^{q_k}$

where $N < q_1 < ... < q_k$ and $N < r_1 < ... < r_k$. Then, $|X_{9} - X_{r}| \le |X_{9} - X(m/2^{-n^{2}})|$ $+ | \times (m / 2^{-n^{+}}) - X_{r} |$ $N-9.$ $\geq \sum_{i=1}^{k} 2^{-q_i} = \sum_{i=1}^{k} 2^{-r_i}$ $N < r₁$ $\leq 2 \sum_{n=n^+}^{\infty} 2^{n^{\gamma}}$ $2^{q_i} < 2^{-n_i+1}$ $2^{-r_1} < 2^{-n^2}$ $\frac{2}{1-2^{6}}$ 2^{n+8} $\frac{1}{2}$ (a) max $\frac{4}{1-2}$ 14-r)