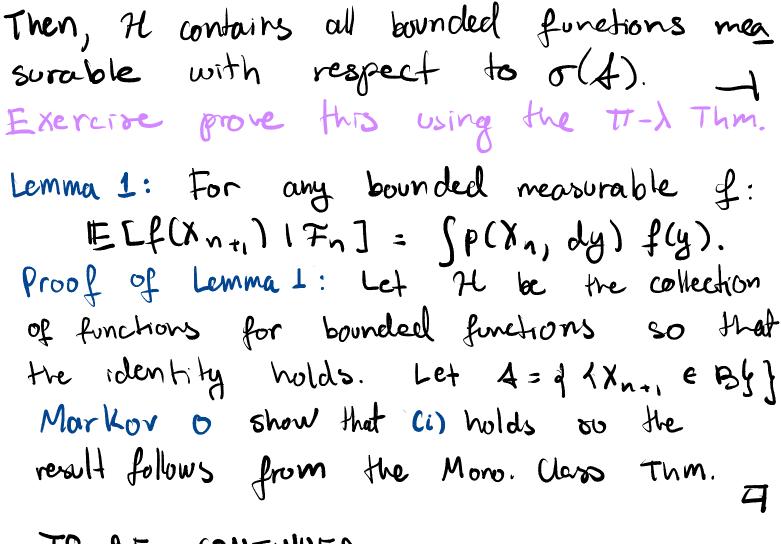
Last time Today D'Intro to Markov Chains D Formal construction o Formal Construction | D Markov Property. For mal construction continued Recall that given a measure for Xo 1 we defined a measure over (s", S") $P(\chi_j \in \Theta_j, 0 \le j \le n) =$ on $(\Omega_0, \mathcal{Z}_0) = (S^N, S^N)$ Note that this construction yields measures for each xES via n=Sx, we use $P_x = P_{\delta_x}$. Further, for an arbitrary M P_M(A) = S_M(dx) P(A). Our goal today is to prove several

versions of Markov's Property. Recall Xn(w) = wn. Theorem (Markovo) Xn is a Markov Chain with respect to $\mathcal{F}_n = \sigma(X_0, X_1, ..., X_n)$ with transition transition probability P, i.e., $P(X_{n+1} \in B \mid \mathcal{F}_n) = P(X_n, B).$ Proof: We show that p(Xn, B) is a version of E[41xmiebs [Fn]. p(xn, B) is clearly Fn measurable. Let A= 1 XoeB, XEB1,..., XneBny, Bn+1 = B. By definition $\int 1_{1\times n+1} e_{\theta_{y}} dP_{m} = P_{m}(A \cap \{\times_{n+1} \in B_{\theta}\})$ = $\int \mu (dx_0) \int p(x_0, dx_1) \dots \int p(x_n, dx_n) p(x_n, B)$ Bo Bi We would like to say that (?) = Sp(Xnn, B) d Pm. To do so we follow a standard pipetine, role that for any CES $\int_{A} \mathcal{L}_{c}(X_{n}) dH_{m} = \int_{B_{0}} \mu(dx_{0}) \cdots \int_{a} p(x_{n-\mu} dx_{n}) \mathcal{L}_{c}(X_{n}).$ Then, we have cquality for simple functions

and by BCT the equality is valid
for bounded measurable functions and so
(?) follows.
A simple computation reveals that
the set A st.
J I (Xn+1) EBY dPA = Jp(Xn+1), B) dPa (ST)
forms a 1-system. Moreover, we proved
that this equality holds for A=1×0680,..., Xn68n3
which forms a TT-system. By the TT-X
Theorem (Thm 2.1.6 in Durrett) equality
(St) holds #A 62n, which proves the
rebult.
Next we prove a couple of extensions
of the Markov Property where II (Xn+1) CBJ
is substituted by a bounded form of the
future, h(Xn, Xn+1,...).
Let
$$\Theta_m: \Omega_0 \to \Omega_0$$

given by
 $\Theta_m(Workov I)$ Let Y: $\Omega_0 \to R$ be bounded



TO BE CONTINUED ...